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Asymptotic framework for flood models comparison

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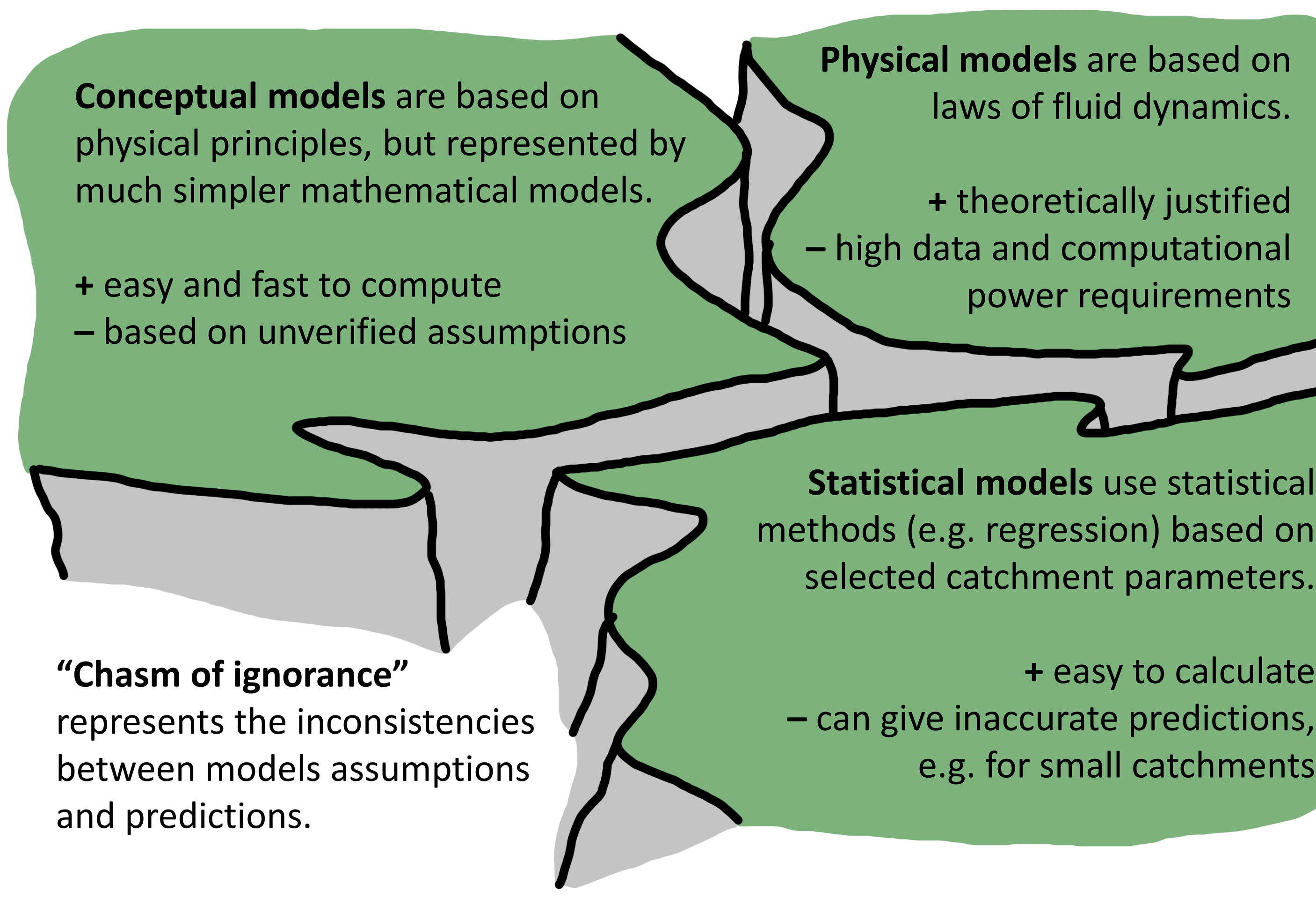


Fundamental problems of flood estimation

The fundamental problem of hydrology is to predict river flow given the time series of rainfall data. The state-of-the-art approaches being used include physical, conceptual, and statistical modelling. Despite their overall good performance, it is observed that data-based modelling approaches in some situations give inaccurate predictions, especially in conditions underrepresented in the training data [Keith, 2019]. Understanding the limits of model applicability remains an open challenge.

> *"(…) advancing the science of hydrology will require not only developing theories that get the right answers but also testing whether they get the right answers for the right reasons."*
>
> James W. Kirchner (2006)

Here we present a unified framework using asymptotic analysis, which highlights differences between these modelling approaches. The framework provides clear analytical and numerical benchmarks on the different approaches in a range of scenarios. Consequently, the proposed approach may lead to better understanding of the uncertainties in hydrologic models, and development of more theoretically-justified flood estimation methods.

How can mathematical modelling and asymptotic analysis help to unify catchment models?

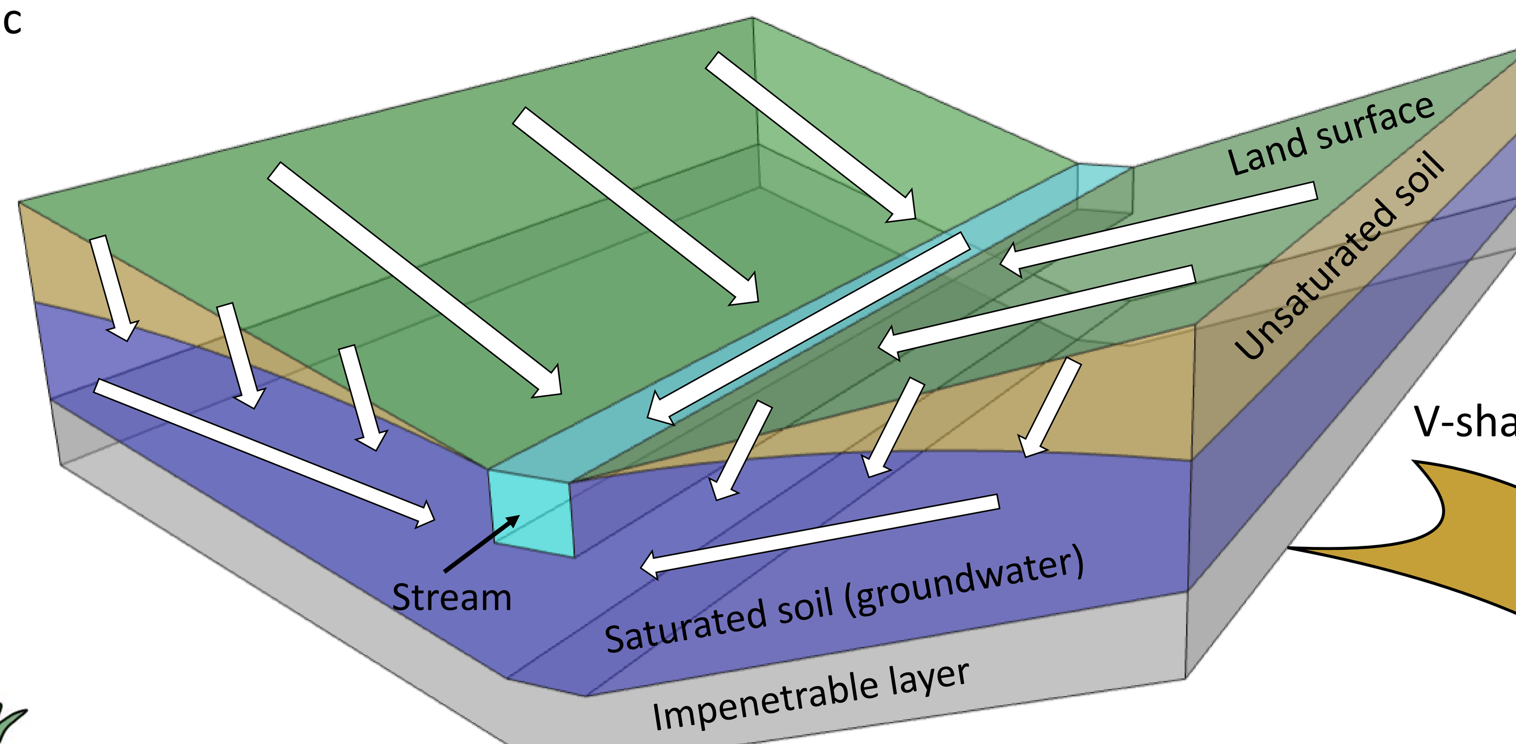


Fig 1. Simplified 3D V-shape catchment model

STEP 1. Instead of numerical analysis on specific catchments we consider a **simplified catchment geometry** (e.g. V-shape catchment) reflecting the general properties of real-world systems (Fig 1).

STEP 2. Nondimensionalisation of full 3D model allows us to extract **key elements responsible for generating flow**. The model can be reduced to a 2D model (Fig 2) and in certain scenarios to a coupled set of 1D flow submodels (e.g. representing groundwater and overland flow).

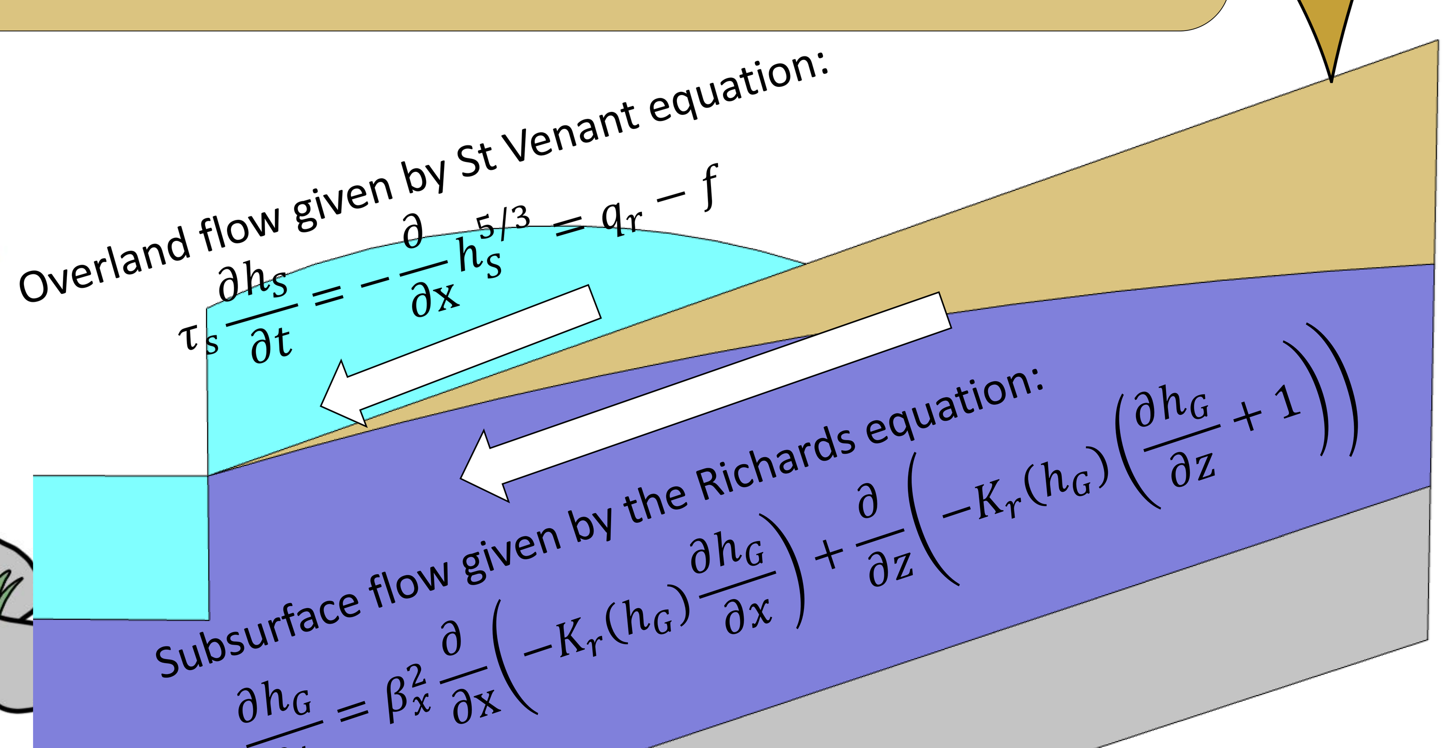


Fig 2. Reduced 2D catchment cross-section

Key model 2D model dimensionless parameters:

$\beta_x = \frac{L_z}{L_x} \sim 10^{-3}$	aspect ratio of cross section along hillslope
$\tau_s = \frac{L_s}{t_0 r} \sim 10^{-1}$	ratio of overland and groundwater timescale
$q_r = \frac{r}{K_s} \sim 10^{-1}$	ratio of rainfall and hydraulic conductivity

STEP 3. Reduced physical models are analysed using asymptotic methods in order to **extract key scaling laws**. Our analysis shows different scaling laws are predicted under different models and highlights possible sources of inter-model inaccuracy.

Consider the Boussinesq equation for the groundwater height H on a hillslope:

$$S_{y.a.}\frac{\partial H}{\partial t} = \frac{\partial q}{\partial x} + r \qquad \text{where} \qquad q = K_s\left(H\frac{\partial H}{\partial x}\cos\theta + H\sin\theta\right)$$

We have showed that in short time asymptotic:

$$Q = q(x=0,t) = Q_0 + (q_{in} - q_0)\sqrt{\frac{K_s h_r \cos\theta}{\pi S_{y.a.}}}\, t^{1/2} + O(t)$$

Currently used conceptual models (e.g. [Bell, 2007]) use linear equations for the groundwater flow, with the following short time asymptotic:

$$Q = Q_0 + c(q_{in} - q_0)t + O(t^2)$$

where c is a parameter fitted to the data.

The comparison of asymptotic behaviour leads to two important conclusions:

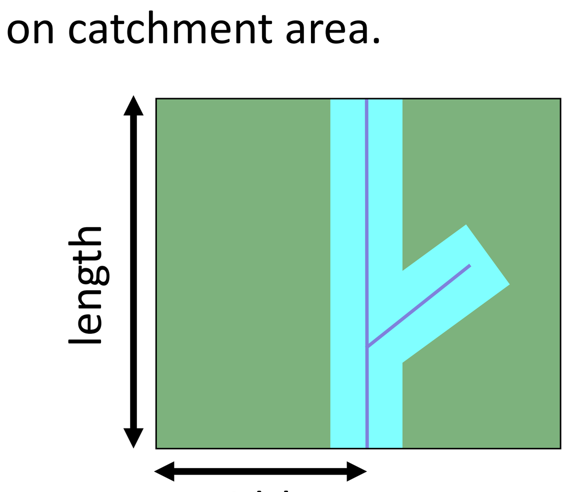
1) These models give different scaling at small times ($t^{1/2}$ vs t); the conceptual model does not accurately predict flow in our simplified geometry.
2) Asymptotic analysis demonstrates the fitted parameters of conceptual models can be related to physical properties of the catchment ($S_{y.a.}$, h_r, θ).

STEP 4. The observed inconsistencies in scaling laws may provide information on how to **improve existing flood estimation methods**. Since scaling laws are derived based on simplified geometries, any model modifications need to be verified on real-world data.

An important result from our work, based on analysis of physical models, is that flow produced by rainfall of typical duration is proportional to the catchment length. In a statistical model currently used by the UK Environmental Agency [Kjeldsen, 2008], it is assumed that flow is dependent on catchment area.

We have modified the statistical procedure to use the catchment (river) length instead of area and fitted it to gauged catchments in UK (data source: NRFA). This allowed us to:

1) confirm the predictions of physical model,
2) slightly improve the model performance,
3) significantly improve results of the model diagnostic (e.g. QQ plot).



Conclusions and further directions

We clearly demonstrated that underlaying assumption of conceptual and statistical model leads to different catchment behaviour than predicted by benchmark physical models. As a consequence, data-based models may give inaccurate predictions when applied in situations underrepresented in the training data. As demonstrated for a particular statistical model, we can use asymptotic results for physical models to improve currently used methods.

These are preliminary results of a long-term project, in which we aim to provide hydrologists with new mathematical tools, insights, and more theoretically-justified catchment models. The important milestones of the project are:

1. Extracting key dependencies between model variables for the full 2D/3D catchment model.
2. Using asymptotic results to develop theoretically-justified computational models for situations not captured accurately by the standard methods.
3. Extending benchmark scenarios by including most recent advances in hydrology literature.

References:

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